

# Radical Conservatism And Nucleon Decay

Frank Wilczek<sup>a \*</sup>

<sup>a</sup>Institute for Advanced Study,  
School of Natural Sciences, Olden Lane,  
Princeton, New Jersey 08540

Unification of couplings, observation of neutrino masses in the expected range, and several other considerations confirm central implications of straightforward gauge unification based on  $SO(10)$  or a close relative and incorporating low-energy supersymmetry. The remaining outstanding consequence of this circle of ideas, yet to be observed, is nucleon instability. Clearly, we should aspire to be as specific as possible regarding the rate and form of such instability. I argue that not only esthetics, but also the observed precision of unification of couplings, favors an economical symmetry-breaking (Higgs) structure. Assuming this, one can exploit its constraints to build reasonably economical, overconstrained yet phenomenologically viable models of quark and lepton masses. Putting it all together, one arrives at reasonably concrete, hopeful expectations regarding nucleon decay. These expectations are neither ruled out by existing experiments, nor hopelessly inaccessible.

Radical conservatism, in the sense of Wheeler, is the doctrine of taking good successful ideas seriously, and pressing them hard, to see if they break. It has a noble history, for example in quantum electrodynamics. Here I will follow this philosophy for straightforward gauge unification. In the recent literature many more exotic ideas about physics beyond the standard model have been explored [1], and there is nothing wrong with that, but one should not forget that the simplest possibilities, already broadly envisioned by the early 80s, have not been disproved. Quite the contrary. For reasons I will presently summarize, I believe that after years of marvelous precision work at LEP and elsewhere, the discovery of non-zero neutrino mass at SuperK, and the non-discovery of any among a plethora of suggested exotica, the early ideas look better than ever. Maybe it is a coincidence – excuse me, a series of coincidences – or a conspiracy. Maybe. But I doubt it, and so should you.

This is not to say that gauge theory unification is the end of all desire, or a Theory of Everything. It certainly is not. Even if true, it leaves many

loose ends and unanswered questions. But if true it represents a worthy addition to the Standard Model, a major additional insight into Nature, and a foundation for further progress.

And, most fortunately, gauge theory unification is quite concretely a theory of Something. In many ways the crown jewel among its predictions is that nucleons should decay. The possibility of such decay directly reflects the unity of matter – interconvertibility of quarks and leptons – and connects to the cosmological asymmetry between matter and antimatter. The quest to observe nucleon decay has already inspired heroic, though so far fruitless, experimental efforts.

Actually, after a moment's reflection, I want to take back that 'so far fruitless'. Creative efforts to observe nucleon decay have led, through the great IMB, Kamiokande, and SuperK lineage of experiments, to technology that has proved immensely fruitful for neutrino physics. Highlights include observation of the supernova 1987a burst, observation of oscillations in neutrinos deriving from atmospheric cosmic rays [2], and observation of a non-zero but anomalous high-energy solar neutrino flux – each of these representing an achievement of historic proportions. And even the negative result of nucleon instability searches to date has been of genuine positive value. It

---

\*Invited Talk at NNN99 Workshop, September 23-25, 1999, at SUNY-Stony Brook. IASSNS-HEP 00/09. Research supported in part by DOE grant DE-FG02-90ER40542.

provided an early motivation for supersymmetric unification, and continues to offer powerful guidance as to what proposals for physics beyond the standard model can be considered plausible.

In any case, we are gathered here to consider whether still more heroic, not to mention expensive, efforts in this direction are warranted. And I want to argue as forcefully as I can for what I believe, that they most certainly are. For upon putting together a number of elegant, successful ideas one arrives at reasonably concrete, hopeful expectations regarding nucleon decay, as I shall indicate. These expectations are neither hopelessly inaccessible, nor ruled out by existing experiments. Furthermore, the branching fractions can discriminate among different possibilities for physics at the unification scale. I will be drawing on results from a recent long, numerically dense analysis by K. Babu, J. Pati, and myself [3]. This work in turn draws on an extensive previous literature; for details and references you should refer to our paper.

## 1. The Case for Unification

The argument for gauge unification is powerful and many-faceted. I will review it in seven installments, starting with the strongest and working down:

1. the unification of quantum numbers and multiplets;
2. the unification of couplings, using supersymmetry;
3. the explanation of small neutrino masses, in the observed range;
4. the explanation of the  $b/\tau$  mass ratio;
5. explaining why things that might otherwise happen, do not;
6. propinquity of the unification and quantum gravitational scales;
7. broad consistency with string/M theory.

### 1.1. quantum numbers and multiplets

The standard model of particle physics is based upon the gauge groups  $SU(3) \times SU(2) \times U(1)$  of strong, electromagnetic and weak interactions acting on the quark and lepton multiplets as shown in Figure 1.

In this Figure I have depicted only one family ( $u, d, e, \nu_e$ ) of quarks and leptons; in reality there seem to be three families that are mere copies of one another as far as their interactions with the gauge bosons are concerned, but differ in mass. Actually in the Figure I have ignored masses altogether, and allowed myself the convenient fiction of pretending that the quarks and leptons have a definite chirality – right- or left-handed – as they would if they were massless. The more precise statement, of course, is that the gauge bosons couple to currents of definite chirality. The chirality is indicated by a subscript R or L. Finally the little number beside each multiplet is its assignment under the  $U(1)$  of hypercharge, which is the average of the electric charge of the multiplet.

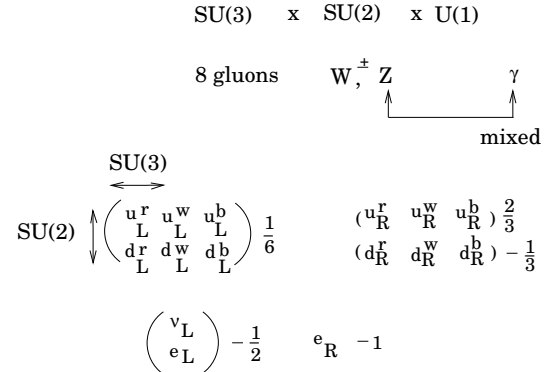


Figure 1. The gauge groups of the standard model, and the fermion multiplets with their hypercharges.

While little doubt can remain that the Standard Model is essentially correct, a glance at Figure 1 is enough to reveal that it is not a complete or final theory. To remove its imperfections, while building upon its solid success, is a worthy challenge.

Given that the strong interactions are governed by transformations among three colors, and the weak by transformations between two others, what could be more natural than to embed both theories into a larger theory of transformations among all five colors?

This idea has the additional attraction that an extra  $U(1)$  symmetry commuting with the strong  $SU(3)$  and weak  $SU(2)$  symmetries automatically appears, which we can attempt to identify with the remaining gauge symmetry of the standard model, that is hypercharge. For while in the separate  $SU(3)$  and  $SU(2)$  theories we must throw out the two gauge bosons which couple respectively to the color combinations  $R+W+B$  and  $G+P$ , in the  $SU(5)$  theory we only project out  $R+W+B+G+P$ , while the orthogonal, traceless combination  $(R+W+B)-\frac{3}{2}(G+P)$  remains.

Finally, the possibility of unified gauge symmetry breaking is plausible by analogy; after all, we know for sure that gauge symmetry breaking occurs in the electroweak sector.

Georgi and Glashow [4], (and in a different way, Pati and Salam [5]) showed how these ideas can be used to bring some order to the quark and lepton sector, and in particular to supply a satisfying explanation of the weird hypercharge assignments in the standard model. As shown in Figure 2, the five scattered  $SU(3) \times SU(2) \times U(1)$  multiplets get organized into just two representations of  $SU(5)$ .

In making this unification it is necessary to allow transformations between (what were previously considered to be) particles and antiparticles, and also between quarks and leptons. It is convenient to work with left-handed fields only. Since the conjugate of a right-handed field is left-handed, we don't lose anything by doing so – though we must shed traditional prejudices about a rigorous distinction between matter and anti-matter, since these get mixed up. Specifically, it will not be possible to declare that matter is what carries positive baryon and lepton number, since the unified theory does not conserve these quantum numbers.

As shown in Figure 2, there is one group of ten left-handed fermions that have all possible combinations of one unit of each of two different colors, and another group of five left-handed fermions

that each carry just one negative unit of some color. These are the ten-dimensional antisymmetric tensor and the complex conjugate of the five-dimensional vector representation, commonly referred to as the five-bar. In this way, *the structure of the standard model, with the particle assignments gleaned from decades of experimental effort and theoretical interpretation, is perfectly reproduced by a simple abstract set of rules for manipulating symmetrical symbols*. Thus for example the object  $RB$  in this Figure has just the strong, electromagnetic, and weak interactions we expect of the complex conjugate of the right-handed up-quark, without our having to instruct the theory further.

A most impressive, though simple, exercise is to work out the hypercharges of the objects in Figure 2 and checking against what you need in the Standard Model. These ugly ducklings of the Standard Model have matured into quite lovely swans.

In addition to the conventional quarks and leptons the  $SO(10)$  spinor contains an additional particle, an  $SU(3) \times SU(2) \times U(1)$  singlet. (It is even an  $SU(5)$  singlet.) Usually when a theory predicts unobserved new particles they are an embarrassment. But these  $N$  particles – there are three of them, one for each family – are a notable exception. Indeed, they are central to the emerging connection between neutrino masses and unification, as I shall discuss below.

## 1.2. unification of couplings using supersymmetry

We have just seen that simple unification schemes are spectacularly successful at the level of classification. New questions arise when we consider dynamics.

Part of the power of gauge symmetry is that it fully dictates the interactions of the gauge bosons, once an overall coupling constant is specified. Thus if  $SU(5)$  or some higher symmetry were exact, then the fundamental strengths of the different color-changing interactions would have to be equal, as would the (properly normalized) hypercharge coupling strength. In reality the coupling strengths of the gauge bosons in

SU(5): 5 colors RWBGP  
10: 2 different color labels (antisymmetric tensor)

$$\begin{array}{lll} u_L : & RP, & WP, & BP \\ d_L : & RG, & WG, & BG \\ u_L^c : & RW, & WB, & BR \\ e_L^c : & GP & & \end{array} \begin{pmatrix} 0 & u^c & u^c & u & d \\ & 0 & u^c & u & d \\ & & 0 & u & d \\ & * & & 0 & e \\ & & & & 0 \end{pmatrix}$$

$\bar{5}$ : 1 anticolor label

$$\begin{array}{lll} d_L^c : & \bar{R}, & \bar{W}, & \bar{B} \\ e_L : & \bar{P} & & (d^c \ d^c \ d^c \ e \ \nu) \\ \nu_L : & \bar{G} & & \end{array}$$

$$\boxed{Y = -\frac{1}{3} (R+W+B) + \frac{1}{2} (G+P)}$$

Figure 2. Unification of fermions in SU(5) There is a beautiful extension of SU(5) to the slightly larger group SO(10). With this extension, one can unite all the observed fermions of a family, plus one more, into a *single* multiplet [6]. The relevant representation for the fermions is a 16-dimensional spinor representation. Some of its features are depicted in Figure 3.

$$\begin{array}{l} \text{SO(10): 5 bit register} \\ (\pm \pm \pm \pm \pm) : \text{even \# of -} \\ \begin{array}{ll} (+ + - | + -) & 6 \quad (u_L, d_L) \\ 10 : (+ - - | + +) & 3 \quad u_L^c \\ & (+ + + | - -) \quad 1 \quad e_L^c \end{array} \\ \begin{array}{ll} \bar{5} : (+ - - | - -) & \bar{3} \quad d_L^c \\ & (- - - | + -) \quad \bar{2} \quad (e_L, \nu_L) \end{array} \\ 1 : (+ + + | + +) \quad 1 \quad N_R \end{array}$$

Figure 3. Unification of fermions in SO(10). The rule is that all possible combinations of 5 + and - signs occur, subject to the constraint that the total number of - signs is even. The SU(5) gauge bosons within SO(10) do not change the numbers of signs, and one sees the SU(5) multiplets emerging. However there are additional transformations in SO(10) but not in SU(5), which allow any fermion to be transformed into any other.

SU(3)×SU(2)×U(1) are not observed to be equal, but rather follow the pattern  $g_3 \gg g_2 > g_1$ .

Fortunately, experience with QCD emphasizes that couplings *run* [8]. The physical mechanism of this effect is that in quantum field theory the vacuum must be regarded as a polarizable medium, since virtual particle-antiparticle pairs can screen charge. For charged gauge bosons, as arise in non-abelian theories, the paramagnetic (antiscreening) effect of their spin-spin interaction dominates, which leads to asymptotic freedom. As Georgi, Quinn, and Weinberg pointed out [9], if a gauge symmetry such as SU(5) is spontaneously broken at some very short distance then we should not expect that the effective couplings probed at much larger distances, such as are actually measured at practical accelerators, will be equal. Rather they will all have been affected to a greater or lesser extent by vacuum screening and anti-screening, starting from a common value at the unification scale but then diverging from one another. The pattern  $g_3 \gg g_2 > g_1$  is just what one should expect, since the antiscreening effect of gauge bosons is more pronounced for larger gauge groups.

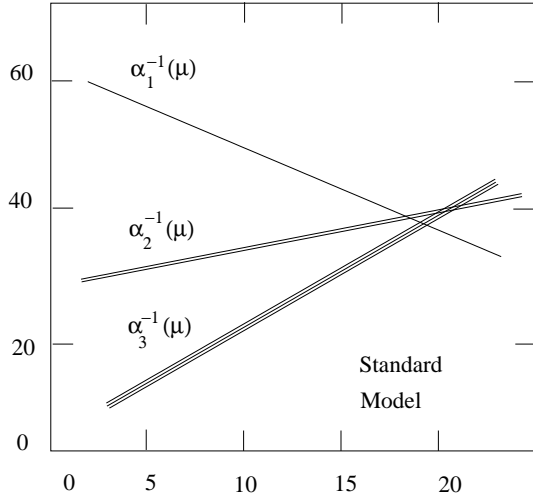


Figure 4. The failure of the running couplings, normalized according to  $SU(5)$  and extrapolated taking into account only the virtual exchange of the “known” particles of the standard model (including the Higgs boson) to meet. Note that only with fairly recent experiments [7], which greatly improved the precision of the determination of low-energy couplings, has the discrepancy become significant.

The running of the couplings gives us a truly quantitative handle on the ideas of unification. To specify the relevant aspects of unification, one basically needs only to fix two parameters: the scale at which the couplings unite, (which is essentially the scale at which the unified symmetry breaks), and their common value when they unite. Given these, one calculates three outputs, the three *a priori* independent couplings for the gauge groups in  $SU(3) \times SU(2) \times U(1)$ . Thus the framework is eminently falsifiable. The astonishing thing is, how close it comes to working (See Figure 4).

The GQW calculation is remarkably successful in explaining the observed hierarchy  $g_3 \gg g_2 > g_1$  of couplings and the approximate stability of the proton. In performing it, we assumed that the known and confidently expected particles of the standard model exhaust the spectrum up to the unification scale, and that the rules of quantum field theory could be extrapolated without alter-

ation up to this mass scale – thirteen orders of magnitude beyond the domain they were designed to describe. It is a triumph for minimalism, both existential and conceptual.

On closer inspection, however, it is not quite good enough. Accurate modern measurements of the couplings show a small but definite discrepancy between the couplings, as appears in Figure . 4. And heroic dedicated experiments to search for proton decay at the rate expected from exchange of the additional gauge bosons present in  $SU(5)$  but not in the Standard Model did not find it [10]. They currently exclude the minimal  $SU(5)$  prediction  $\tau_p \sim 10^{31}$  yrs. by about two orders of magnitude.

If we just add particles in some haphazard way things will only get worse: minimal  $SU(5)$  nearly works, so a generic perturbation will be deleterious. Even if some *ad hoc* prescription could be made to work, that would be a disappointing outcome from what appeared to be one of our most precious, elegantly straightforward clues regarding physics well beyond the Standard Model.

Fortunately, there is a compelling escape from this impasse. That is the idea of supersymmetry [11]. Supersymmetry is certainly not a symmetry in nature: for example, there is certainly no bosonic particle with the mass and charge of the electron. However there are several reasons for thinking that supersymmetry might be spontaneously, and only relatively mildly broken, so that the superpartners are no more massive than  $\approx 1$  Tev. The most concrete arises in calculating radiative corrections to the  $(\text{mass})^2$  of the Higgs particle from diagrams of the type shown in Figure 5. One finds that they make an infinite, and also large, contribution. By this I mean that the divergence is quadratic in the ultraviolet cutoff. No ordinary symmetry will make its coefficient vanish. If we imagine that the unification scale provides the cutoff, we will find, generically, that the radiative correction to the  $(\text{mass})^2$  is much larger than the total value we need to match experiment. This is an ugly situation.

In a supersymmetric theory, if the supersymmetry is not too badly broken, it is possible to do better. For any set of virtual particles that might circulate in the loop there will be another

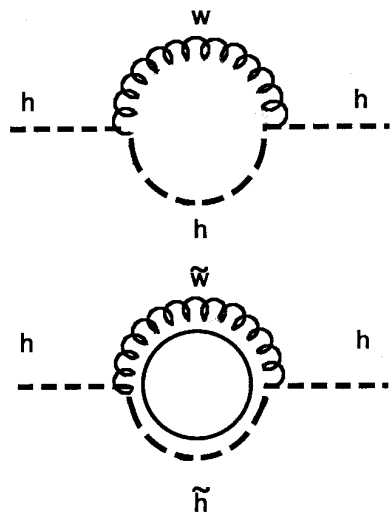


Figure 5. Contributions to the Higgs field self-energy. These graphs give contributions to the Higgs field self-energy which separately are formally quadratically divergent, but when both are included the divergence is removed. In models with broken supersymmetry a finite residual piece remains. If one is to obtain an adequately small finite contribution to the self-energy, the mass difference between Standard Model particles and their superpartners cannot be too great. This – and essentially only this – motivates the inclusion of virtual superpartner contributions in Figure 6 beginning at relatively low scales.

graph with their supersymmetric partners circulating. If the partners were accurately degenerate, the contributions would cancel. Taking supersymmetry breaking into account, the threatened quadratic divergence will be cut off only at virtual momenta such that the difference in  $(\text{mass})^2$  between the virtual particle and its supersymmetric partner is negligible. Notice that we will be assured adequate cancellation if and only if supersymmetric partners are not too far split in mass – in the present context, if the splitting times the square root of the fine structure constant is not much greater than the weak scale.

The effect of low-energy supersymmetry on the

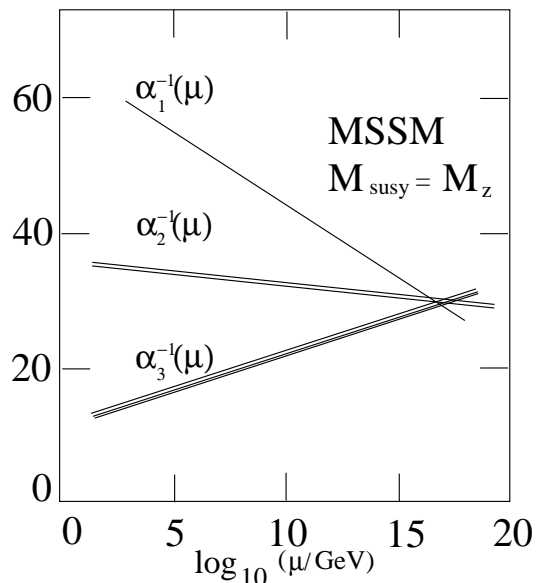


Figure 6. When the exchange of the virtual particles necessary to implement low-energy supersymmetry, a calculation along the lines of Figure 4 comes into adequate agreement with experiment.

running of the couplings was first considered long ago [12], in advance of the precise measurements of low-energy couplings or of the modern limits on nucleon decay. One might have feared that such a huge expansion of the theory, which essentially doubles the spectrum, would utterly destroy the approximate success of the minimal  $SU(5)$  calculation. This is not true, however. To a first approximation since supersymmetry is a space-time rather than an internal symmetry it does not affect the group-theoretic structure of the calculation.

Thus to a first approximation the absolute rate at which the couplings run with momentum is affected, but not the relative rates. The main effect is that the supersymmetric partners of the color gluons, the gluinos, weaken the asymptotic freedom of the strong interaction. Thus they tend to make its effective coupling decrease and approach the others more slowly. Thus their merger re-

quires a longer lever arm, and the scale at which the couplings meet increases by an order of magnitude or so, to about  $10^{16}$  GeV. An immediate effect of raising the scale is to raise the mass of the gauge bosons that can mediate proton decay, so that the experimental bounds are no longer contradicted. (On nucleon stability, more below.)

I want to emphasize that this very large new mass scale has emerged unforced from the internal logic of the Standard Model itself. It will appear in several of our further considerations, and so for later reference let's give it a name, the unification scale, and the token  $M_U$ .

Since the running of the couplings with scale is logarithmic, the unification of couplings calculation is not terribly sensitive to the exact scale at which supersymmetry is broken, say between 100 GeV and 10 TeV. It is a result robust, at the few *per cent* level, against uncertainties of this sort. This robustness is fortunate (and virtually unique among the phenomenological signatures of supersymmetry), because at present the mechanism of supersymmetry breaking, and therefore the spectrum of sfermions and gauginos, is quite uncertain. The unification of couplings is also robust against addition of additional particles, so long as they come in complete, approximately degenerate  $SU(5)$  multiplets. Additional uncertainties do arise from the details of the unified symmetry breaking at the high scale. I'll discuss these further below. The main conclusion is that these corrections are also at the few *per cent* level, so long as the symmetry breaking is implemented economically (*i.e.*, using the simplest Higgs field representations).

On the other hand, our successful unification of couplings calculation is most definitely *not* robust against radical changes in its embedding framework, such as abandoning low-energy supersymmetry, using radically different unification groups, or allowing the virtual particles to wander off into extra dimensions. If any of these ideas are correct, the spectacular existing agreement of theory and experiment, displayed in Figure 6, would seem to be a 'coincidence' – imputing to Mother Nature a rather sadistic propensity to tease.

### 1.3. neutrino mass

It is important to realize that the degrees of freedom of the Standard Model permit neutrino masses. A minimal implementation of the construction requires an interaction of the type

$$\Delta\mathcal{L} = \kappa_{ij} L^{\alpha ai} L^{\beta bj} \epsilon_{\alpha\beta} \phi_a^\dagger \phi_b^\dagger + \text{h.c.} , \quad (1)$$

where  $i$  and  $j$  are family indices;  $\kappa_{ij}$  is a symmetric matrix of coupling constants; the  $L$  fields are the left-handed doublets of leptons, with Greek spinor indices, early Roman weak  $SU(2)$  indices, and middle Roman flavor indices; and finally  $\phi$  is the Higgs doublet, with its weak  $SU(2)$  index. Two-component notation has been used for the spinors, to emphasize that this way of forming mass terms, although different from what we are used to for quarks and charged leptons, is in some sense more elementary mathematically.  $\Delta\mathcal{L}$  becomes a neutrino mass term when the  $\phi$  field is replaced by its vacuum expectation value  $\langle\phi^a\rangle = v\delta_1^a$ .

Although this Eq. (1) is a possible interaction for the degrees of freedom in the Standard Model, it is usually considered to be *beyond* the Standard Model, for a very good reason. The new term differs from the terms traditionally included in the Standard Model in that the product of fields has mass dimension 5, so that the coefficient  $\kappa$  must have mass dimension -1. In the context of quantum field theory, it is a nonrenormalizable interaction. When one includes it in virtual particle loops, one will find amplitudes containing the dimensionless factors of the type  $\kappa\Lambda$ , where  $\Lambda$  is an ultraviolet cutoff. In this framework, therefore, one cannot accept  $\Delta\mathcal{L}$  as an elementary interaction. It can only be understood within a larger theoretical context.

Given a numerical value for the neutrino mass, we can infer a scale beyond which  $\Delta\mathcal{L}$  cannot be accurate, and degrees of freedom beyond the Standard Model must open up. To get oriented, let us momentarily pretend that  $\kappa$  is simply a number instead of a matrix, and that  $m = 10^{-2}$  eV is the neutrino mass. Then, using  $v = 250$  GeV for the vacuum expectation value, we calculate

$$1/M \equiv \kappa = m/v^2 = 1/(6 \times 10^{15} \text{ GeV}) . \quad (2)$$

When energy and momenta of order  $M$  begin to circulate in loops the form of the interaction must be modified. Otherwise the dangerous factor  $\kappa\Lambda$  will become larger than unity, inducing large and uncontrolled radiative corrections to all processes, and rendering the success of the Standard Model accidental.

Thus we trace the “absurdly small” value of the observed neutrino mass scale to an “absurdly large” fundamental mass scale. You will not fail to notice that the new scale we infer here, directly from the observed value of the neutrino mass is, quantitatively, none other than  $M_U$ . This is most definitely not a coincidence, as I’ll now explain.

Let us return to the question of the  $N$  masses. Because the  $N^i$  are singlets, mass terms of the type

$$\Delta\mathcal{L}_N = \eta_{ij} N^{\alpha i} N^{\beta j} \epsilon_{\alpha\beta} \quad (3)$$

with  $\eta_{ij}$  a symmetric coupling matrix, are consistent with  $SU(3) \times SU(2) \times U(1)$  symmetry. This term of course greatly resembles the effective interaction responsible for neutrino masses, Eq. (1), but conceptually the difference is crucial. Because the  $N$ s are Standard Model singlets the Higgs doublets that occurred in Eq. (1) need not appear here. A consequence is that the operators appearing in Eq. (3) have mass dimension 3, so that the  $\eta_{ij}$  must have mass dimension +1. This interaction therefore does not bring in any ultraviolet divergence problems.

What sets the scale for  $\eta$ ? Although Eq. (3) is consistent with Standard Model gauge symmetries, or even  $SU(5)$ , it is not consistent with  $SO(10)$ . Indeed for the product of spinor 16 we have the decomposition  $16 \times 16 = 10 + 120 + 126$ , where only the 126 contains an  $SU(5)$  singlet component. The most straightforward possibility for generating a term like Eq. (3) in the full theory is therefore to include a Higgs 126, and a Yukawa coupling of this to the 16s. If the appropriate components of the 126 acquire vacuum expectation values, Eq. (3) will emerge. The 126 is a five-index self-dual antisymmetric tensor under  $SO(10)$ , which may not be to everyone’s taste. Alternatively, one can imagine that more complicated interactions, containing products of several simpler Higgs fields which condense, are respon-

sible. These need not be fundamental interactions (they are, of course, non-renormalizable), but could arise through loop effects or by integrating out heavier particles even in a renormalizable field theory.

At this level there are certainly many more options than constraints, so that without putting the discussion of  $N$  masses in a broader context, and making some guesses, one can’t be very specific or quantitatively precise. Nevertheless, I think it is fair to say that these general considerations strongly suggest that  $\eta$  is associated with breaking of unified symmetries down to the Standard Model. Thus, if the general framework is correct, the expected scale for its entries is set by the one we met in the unification of couplings calculation, i.e.  $\eta \sim M_U = 10^{16}$  GeV.

The  $N$ s communicate with the familiar fermions through the Yukawa interactions

$$\Delta\mathcal{L}_{N-L} = g_j^i \bar{N}_i L^{aj} \phi_a^\dagger + \text{h.c.} , \quad (4)$$

using the previous notations but now, in this more conventional term, suppressing the Dirac spinor indices. These interactions are of precisely the type that generate masses for the quarks and charged leptons in the Standard Model. If  $N$  were otherwise massless, the effect of Eq. (4) would be to generate neutrino masses, of the same order as ordinary quark and lepton masses. In  $SO(10)$ , indeed, these masses would be related by simple Clebsch-Gordon and renormalization factors of order unity. Fortunately, as we have seen,  $N$  is far from massless.

Indeed,  $N$  is so massive that for purposes of low-energy physics we can and should integrate it out. This is easy to do. The effect of combining Eq. (3) and Eq. (4) and integrating out  $N$  is to generate

$$\Delta\mathcal{L}_{\text{eff.}} = g_i^k g_j^l (\eta^{-1})_{kl} L^{\alpha ai} L^{\beta bj} \epsilon_{\alpha\beta} \phi_a^\dagger \phi_b^\dagger + \text{h.c.} . \quad (5)$$

Thus we arrive back at Eq. (1), with

$$\kappa_{ij} = g_i^k g_j^l (\eta^{-1})_{kl} . \quad (6)$$

This so-called seesaw equation [13] provides a much more precise version of the loose connection between unification scale and neutrino mass we discussed at the outset. There is much uncertainty in the details, since there is no reliable



detailed theory for the  $g_i^k$  nor the  $\eta$ s. But if  $g$  has an eigenvalue of order unity pointing toward the third family (this is suggested by symmetry and the value of the top quark mass, as discussed below), and if we set the scale for  $\eta$  using the logic above, then we get close to  $10^{-2}$  eV for the  $\tau$  neutrino mass, as observed.

On the face of it, then, neutrino mass of the observed magnitude provide an additional confirmation of our well developed, straightforward, minimalist ideas for unification beyond the Standard Model. It also takes us in a pretty direction where we should be delighted to go: toward more complete symmetry, using  $SO(10)$  (or perhaps, as Pati emphasizes, a smaller but still left-right symmetric variant). Within this circle of ideas, neutrino mass of the observed magnitude is a robust consequence. Outside that circle, it becomes another ‘coincidence’.

#### 1.4. $b/\tau$ mass ratio

Within  $SU(5)$ , or any of its extensions, it is natural to expect certain kinds of regularities among their masses, since quarks and leptons are put together in common multiplets. Specifically, the right-handed  $b$  quark (or, better, the left-handed  $\bar{b}$ ) and the left-handed  $\tau$  lepton can be found in a single  $\bar{5}$  multiplet, whereas their oppositely-handed pairs can be found in a single 10 multiplet. If we assume that their masses are generated in the simplest possible way, using a Higgs field in the  $\bar{5}$ , we find a simple relation – in fact, equality – between the masses. Such equality does not hold, of course, of the observed physical masses. But we must remember that – again, in the circle of ideas around minimalist unification – the fundamental equality is between effective Yukawa couplings normalized at  $M_U$ . Just as for the gauge couplings, we must renormalize this prediction down to laboratory scales, taking into account the effect of virtual particles. When this is done – again, in the minimalist framework – one finds striking agreement between prediction and observation.

Within  $SO(10)$ , one obtains (with similar assumptions) in addition a similar relation between the top quark mass and the underlying Dirac mass of the  $\tau$  neutrino (*i.e.*, the off-diagonal entry

in the seesaw mass matrix). This reinforces the estimate of the heaviest neutrino mass presented above, and furthermore associates that mass with the  $\tau$  neutrino.

The luster of these successes in correlating the masses of the heaviest fermion family is somewhat dimmed by the failure of the simplest hypotheses to explain the pattern of lighter fermion masses and mixings. Of course those masses, being smaller, are *a priori* more sensitive to quantitatively small complications, so that predictions for them are intrinsically less robust. The situation is far from desperate, and I’ll say a bit more about it below.

#### 1.5. things that don’t happen

One frequently encounters jeremiads about the danger of assuming lack of complications such as new strongly interacting sectors (technicolor), compositeness, and – recently popular – additional large dimensions, as one extrapolates from observed energy scales to  $M_U$ . Doubtless there are any number of ways that the radically conservative extrapolation of gauge field theory might go wrong. However, one should not discount the observation that similar jeremiads have been voiced for more than twenty years now, while so far no hint of any of the suggested deviations has in fact materialized.

Quite the contrary. As precision measurements of Standard Model parameters have converged on minimal supersymmetric unification of couplings, they have also put severe constraints on these picturesque and intuitively appealing, but speculative and phenomenologically gratuitous, possibilities. Likewise, searches for unconventional sources of CP violation and for effects of neutral flavor-changing interactions have come up empty, and put considerable pressure on any suggestion that the fundamental dynamics associated with non-universal flavor interactions, let alone with dynamics that connects quark and leptons, occurs below a scale of several TeV. Conversely, the idea that a large scale like  $M_U$  characterizes these effects, if less tantalizing, is much safer.

There is a slightly naive, but not completely silly, general consideration worthy of mention here. Any ambitious extension of the Standard

Model sufficient to unify quarks and leptons (including any incarnation of string/M theory) will almost certainly involve violation of baryon number, and therefore at some level nucleon instability. With a scale as large as  $M_U$ , we might – as discussed below – just squeeze by the experimental constraints. If the scale is significantly smaller, that becomes much more difficult.

### 1.6. propinquity of the gravity scale

The value of  $M_U$  is, on the appropriate logarithmic scale, remarkably close to the Planck scale  $M_{\text{Planck}} \sim 10^{19}$  GeV. The Planck scale is the scale at which the classical Einstein description of gravity must break down; concretely it is the energy scale at which exchange of virtual gravitons competes quantitatively with the other interactions. Because  $M_U$  is significantly smaller than the Planck mass, we need not be too nervous about the neglect of quantum gravity corrections to our unification of couplings calculation. Yet because it is not absurdly smaller, we can feel encouraged for the prospect of unification including both gravity and gauge forces, independent of any detailed model.

### 1.7. broad consistency with string/M theory

String/M theory is at present the best candidate framework for incorporating quantum mechanics together with general relativity. Huge challenges remain for construction of scientific world-models on its basis. Specifically, for example, there is no generally accepted understanding of such basic questions as why the macroscopic world looks 3+1 dimensional (whereas the underlying theory is more naturally 9+1 or 10+1 dimensional), nor why the cosmological term is so small, nor even how to formulate either the basic equations or the initial-value problem. The rules of the game have changed over the years, and undoubtedly will continue to do so.

Nevertheless it is intriguing that, given a sense of humor and a bit of good will, one can describe most of the elements of reality utilized in gauge theory unification – the degrees of freedom of the Standard Model, the possibility of low-energy supersymmetry, and enough additional

gauge symmetry for unification of couplings – within string/M theory. The classic (vintage 1984) weakly coupled heterotic phenomenology is mainly concerned with finding solutions to the equations of static classical string theory that reduce well below the Planck scale to effective theories resembling the supersymmetric standard model. (Of course, it is notorious that there are also zillions of apparently equally good solutions that look nothing like our world.) Within this class of models, there is a large subclass that also embodies something close to conventional gauge theory unification. Recent techniques support related constructions at strong coupling [14].

In any case, I think it is certainly fair to say that there is at present no clear contradiction between gauge theory unification as discussed here ‘from the bottom up’, based on straightforward extrapolation of established facts and principles, and string/M theory. This tends to reinforce the significance of our previous, model-independent result  $M_U \sim M_{\text{Planck}}$ .

## 2. Nucleon Decay

### 2.1. supersymmetry and the challenge of exotica

I have argued for the desirability of low-energy supersymmetry based on one major quantitative result (the unification of couplings) and one rather soft theoretical advantage (protection of the weak scale from radiative corrections). Other arguments can and have been made, but I think these two are by far the best, most concrete ones.

Against this less than overwhelming evidence we must weigh considerable complications and several embarrassments.

In the minimal version of the Standard Model (SM), without supersymmetry, one has the possibility of a clean, uniform explanation of the smallness of observed CP violating effects and of neutrino masses, and of the smallness of so far unobserved neutral flavor-changing effects in both the quark and lepton sectors, and of nucleon instability. For given the symmetries and matter content of the minimal SM, all these effects (except CP violation) arise only from higher-dimension, nonrenormalizable interactions. Thus

they appear in the Lagrangian multiplied by coefficients inversely proportional to some mass scale, and if this mass scale is large (say approaching the Planck scale) they represent unobservable, or barely observable, small effects.

CP violation can arise through renormalizable interactions, but only in two special ways. One way is through complicated interference effects involving interference among all three families, as proposed by Kobayashi and Maskawa. The other is through the effects of the notorious  $\theta$  term of QCD. Existing evidence is consistent with the idea that the first of these mechanisms is responsible for all CP violation so far observed; while the  $\theta$  term is, for a reason presumably connected with Peccei-Quinn symmetry and the existence of axions, very small or zero. The adequacy of the minimal SM framework will be tested by future measurements of B-meson properties and searches for elementary electric dipole moments.

As one expands the SM to include supersymmetry this clean, uniform explanation of the absence or smallness of those many diverse species of possible exotica comes undone. Technically, this occurs because the accounting of possible ‘relevant’ (renormalizable, total mass dimension  $\leq 4$ ) interactions is quite different in the supersymmetric case. The bosonic slepton and squark fields have mass dimension unity, as opposed to the fermionic lepton and quark fields, which have mass dimension  $3/2$  (and must appear in pairs within Lorentz invariant candidate interactions), which opens a considerably more capacious Pandora’s box. For example, in the SM without supersymmetry possible baryon number violating interactions have mass dimension at least six, since to make a color singlet they must contain at least three quark fields, and then another fermion (lepton) to make a Lorentz singlet. Using the squark fields, baryon-number violating interactions with dimension 3 can be constructed. Supersymmetry forbids these particular terms (so they may be suppressed by the ratio of supersymmetry breaking to unification scales), but there are several possible supersymmetric dimension 4 and 5 terms. A complementary perspective, looking from the high scale down, is that exchange of heavy fermion partners of scalar or gauge fields brings in propagators

with only one inverse power of the heavy scale, instead of two, and so is less suppressed at low energy. There are also many additional possible sources of CP violation, no longer necessarily involving all three families.

None of these problems appears insurmountable. Indeed, each presents opportunities for theoretical and experimental discovery, and each has generated its own sizable literature. The issue of nucleon instability, in view of its unique sensitivity and deep cosmological significance, may be the most critical and fundamental problem of all, and in the remainder of this talk I will focus on it exclusively. I will be brief, since my collaborators will be covering some of the same ground more thoroughly.

## 2.2. from supersymmetry to Higgsino exchange

The analysis of nucleon instability in supersymmetric theories is difficult to discuss without introducing some technical machinery, since supersymmetry induces some special cancellations which are difficult to see without using superfields. A major result of the analysis is that the possible form of supersymmetric dimension 4 baryon number violating operators is quite restricted, and it is easily forbidden with an appropriate discrete symmetry. A second major result is that the main contribution to dimension 5 baryon number violation comes from Higgsino, not gaugino, exchange.

At first sight it might appear that the move from non-supersymmetric unification, where the leading contributions to nucleon instability arise from dimension 6 operators, suppressed by two inverse powers of the unification scale, to supersymmetric unification, which allows dimension 5 operators and nucleon instability suppressed by only one power of the unification scale, is catastrophic. A number of factors mitigate this crisis, however. The unification scale is somewhat larger, and the relevant Higgsino mass can be larger still; the bottom-line Higgs couplings to the light families are quite small; and one must at the end of the day dress the scalar (squark and slepton) fields appearing in the dimension 5 operators, by exchange of the standard model gaug-

inos, into ordinary quarks and leptons.

Because of all this, in order to obtain a quantitative estimate of nucleon instability one must be quite concrete about masses of the super-heavy Higgs fields and their couplings to ordinary fermions.

### 2.3. doublet-triplet splitting

The Higgs doublet needed for electroweak symmetry breaking in the Standard Model can be embedded in various ways into a representation of the full unified gauge group. The simplest possibility, within  $SU(5)$ , is to embed it within a fundamental, *i.e.*, a **5**. The three extra components form a fractionally charged color triplet. The symmetry instructs us how this triplet couples, and we quickly discover that it is a very dangerous object, because its exchange violates baryon number and destabilizes nucleons. It must be extremely heavy,  $M_{\text{triplet}} \gtrsim 10^{14}$  GeV, in order to be consistent with experimental limits. In particular, it must be very much heavier than its partner, the electroweak Higgs doublet. Theoretically, it is quite challenging to understand how such a large splitting could arise. This is the doublet-triplet splitting problem. Similar problems occur for other unification groups and embeddings.

A profound advantage of supersymmetric unification in  $SO(10)$ , which in my view forms an essential adjunct to its role in protecting the weak scale, is its ability to address the doublet-triplet splitting problem. Over and above its stability to radiative corrections, as mentioned above, there is the question of obtaining the splitting at the classical level in the first place. There are special constraints for the scalar potential due to supersymmetry, arising because it comes, roughly speaking, as the square of a simpler object, the superpotential. They make it possible – in  $SO(10)$ ! – to assure the requisite classical splitting through a simple group-theoretic mechanism [15].

### 2.4. fine structure of coupling unification

By persisting in the radically conservative hypothesis that the striking quantitative success of the unification of couplings calculation, as displayed in Figure 6, is not accidental, we are led to an important conclusion regarding the com-

plexity of unified symmetry breaking.

In general, symmetry breaking effects will split the masses of different components of any Higgs field representation. These splittings lead to logarithmic changes in the running of couplings, as mentioned above. Let us see how they affect the fine structure of coupling constant unification. To lowest (one-loop) order the modifications to the predicted value of the couplings take the form

$$\alpha_i^{-1}(M_Z) = \alpha_U^{-1} - \frac{b_i}{2\pi} \ln(M_Z/M_U) - \Delta_i \quad (7)$$

where

$$\Delta_i = \sum_{\text{submultiplets } \kappa} \frac{-b_i^\kappa}{2\pi} \ln(M_U/m_\kappa). \quad (8)$$

Here  $b_i^\kappa$  is the contribution to the  $i^{\text{th}}$  gauge group  $\beta$  function from the  $\kappa$  submultiplet. If all the  $m_\kappa$  are equal, the effect of the  $\Delta_i$  is merely to renormalize  $M_U$ . In general, however, they will affect the predicted relation among the observed couplings. Suppose that we begin by taking all the  $\Delta_i$  to vanish, which is known to lead to a successful result. Then if we accommodate the perturbations to  $\alpha_1^{-1}$  and  $\alpha_2^{-1}$  by adjusting the two free parameters  $\alpha_U^{-1}$  and  $M_U$ , we are led to alter our prediction for the strong coupling  $\alpha_3^{-1}$  according to

$$\frac{\delta\alpha_3(M_Z)}{\alpha_3(M_Z)^2} = \frac{5}{7}\Delta_1 - \frac{12}{7}\Delta_2 + \Delta_3. \quad (9)$$

Of course, before worrying about possible gratuitous corrections, we must make sure that the basic fields we use to break the unified symmetry down to the standard model give an acceptable zeroth-order answer to begin with. In particular, the contribution of the electroweak doublet, and its triplet partner must be handled carefully. It turns out that if we let the triplet become too heavy, say more than 100 times  $M_U$ , the successful zeroth-order prediction of  $\alpha_3$  becomes endangered. Thus it is impossible to suppress nucleon stability due to this source down to arbitrarily low levels.

Now let us estimate the quantitative impact of different kinds of Higgs structure. If we take a  $5 + \bar{5}$  of  $SU(5)$ , or a 10 of  $SO(10)$ , the result is

$$\frac{\delta\alpha_3(M_Z)}{\alpha_3(M_Z)^2} = \frac{1}{2\pi} \frac{9}{7} \ln(m_3/m_2). \quad (10)$$

If the logarithm is of order unity, this represents a few *per cent* correction to  $\alpha_3(M_Z)$ , which is tolerable.

On the other hand, consider the rank two symmetric traceless tensor 54 of  $SO(10)$ . This is still one of the simpler irreducible representations, but it contains a piece which goes as  $(6, 1, 4/3) + (6, 1, -4/3)$  under the standard model. If this piece is split from its brethren at  $M_U$ , the correction is

$$\frac{\delta\alpha_3(M_Z)}{\alpha_3(M_Z)^2} = \frac{1}{2\pi} \frac{51}{7} \ln(m_3/m_U) , \quad (11)$$

which, for a logarithm of order unity, is in the neighborhood 10-20 %. Uncontrolled corrections of this sort could be expected to upset the applecart. Of course, for more complicated representations, containing more highly charged submultiplets, the situation only gets worse.

At face value, these considerations strongly suggest that the observed success of the unification of couplings can be construed as reassuring confirmation that Nature has good taste: She starts with lots of symmetry, and uses simple, minimalistic symmetry breaking patterns. Of course it's terribly dangerous to rely too heavily on a single number, but we're being radically conservative, and it's taking us where we want to go!

## 2.5. a look toward fermion masses

As we've seen, in supersymmetric unification the leading source of nucleon instability is exchange of superheavy Higgsino fields. In order to pin this down, we must constrain which such fields are present, and how they couple to quarks and leptons. The immediately preceding considerations strongly encourage us to restrict ourselves to the simplest possible field content. For  $SO(10)$ , concretely, this means a small number of adjoints, fundamentals, and spinors.

Having chosen the Higgs content, we must address the coupling to quarks and leptons. There is, of course, a very large amount of data regarding the masses and mixing matrices of quarks and leptons that we should use for guidance. Let me briefly indicate the sorts of considerations that enter, sparing you the hairy details. (Actually,

somewhat to my surprise, things work out rather elegantly, at least for the second and third families.)

In the supersymmetric standard model, which we want to recover at low energy, there are two electroweak doublets. These emerge as the dregs of a mass-generation process that gives superheavy masses  $\sim M_U$  to all the other Higgs fields. In general, these dregs will be made up of bits and pieces coming from different irreducible  $SO(10)$  multiplets, *i.e.* adjoints, fundamentals, and spinors.

Now a fundamental  $10_H$  will couple to the matter 16s by a term of the form  $g_{ij} 16_i \cdot 16_j \cdot 10_H$ , where  $i, j$  are family indices and the  $g_{ij}$  are coupling constants. Group theory requires that  $g_{ij}$  is symmetric in  $i$  and  $j$ . When the  $10_H$  acquires a vacuum expectation value, these couplings will contribute to the observable fermion mass matrices. The group theory also correlates the contributions to different quark and lepton mass matrices.

Similarly an effective coupling of the type  $h_{ij} 16_i \cdot 16_j \cdot 10_H \cdot 45_H$ , involving an adjoint field, can arise. Indeed, to implement a clean gauge symmetry breaking with doublet-triplet splitting we need a very specific form for the vacuum expectation value of  $45_H$  (in the B-L direction). Group theory determines that  $h_{ij}$  is antisymmetric in  $i$  and  $j$ , and the required alignment of the  $45_H$  introduces various factors of 3 (Georgi-Jarlskog [16] factors) into the relative contributions from this term to quark and lepton mass matrices.

By exploiting structures of this sort, and taking guidance from experiment, one can construct remarkably simple and overconstrained, yet not unrealistic, models of quark and lepton masses [3,17,18].

## 2.6. numerical estimates; conclusion

In our long paper, we computed the numerical consequences of a complete model of this sort. In constructing the model we were forced to make several uncertain choices for the Higgs structure and couplings, and in getting to decay rates we were forced to make several further uncertain estimates of SUSY breaking parameters and strong

matrix elements. I wish we could do better. With respect to the microscopic theory I'm afraid the situation is unlikely to improve dramatically any time soon. To make progress, we desperately need to open a dialogue with Nature, through experiment. On the other hand, given sufficient investment in numerical QCD one could improve the estimation of matrix elements. That direction should certainly be pursued, in order to insure that it will be possible to interpret the results properly when and if they do come in.

In any case, doing the best we know how, and with all our cards on the table, Babu, Pati and I by honest toil find

$$\Gamma^{-1}(p) \lesssim 10^{34} \text{ yrs.} \quad (12)$$

within the circle of ideas here advocated. The dominant modes involve strange particles in the final state, and usually (though not necessarily) antineutrinos. The detailed branching ratios, and the nature of the subdominant modes, encode information on additional aspects of unification physics, which is very difficult to access otherwise.

If nucleon instability at these levels were observed it would constitute one of the greatest discoveries in the history of physics, and provide a unique window looking out into the deep structure of physical reality.

## REFERENCES

1. Reviewed in C. Bachas, hep-th/000193 (2000).
2. SuperKamiokande Collaboration, Y. Fukuda et. al., Phys. Rev. Lett. **81**, 1562 (1998).
3. J. Pati, K. Babu, and F. Wilczek, hep-ph/9812538, Nucl. Phys. **B**, in press.
4. H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
5. J.C. Pati and A. Salam, Phys. Rev. Lett. **31**, 661 (1973) and Phys. Rev. **D10**, 275 (1974).
6. H. Georgi, in Particles and Fields, AIP, New York (1975), p. 575 (C.E. Carlson, Ed); H. Fritzsch and P. Minkowski, Ann. Phys. **93**, 193 (1975).
7. LEP Electroweak Working Group, preprint CERN-PPE/96-183 (Dec. 1996).
8. D. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).
9. H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).
10. See for example G. Blewitt, *et al*, Phys. Rev. Lett. **55**, 2114 (1985), and the latest Particle Data Group compilations.
11. A very useful introduction and collection of basic papers on supersymmetry is S. Ferrara, *Supersymmetry* (2 vols.) (World Scientific, Singapore 1986). Another excellent standard reference is N.-P. Nilles, Phys. Reports **110**, 1 (1984).
12. S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981).
13. M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, ed. P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315; T. Yanagida, Proc. of the Workshop on Unified Theory and Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto (KEK, 1979).
14. P. Horava and E. Witten, Nucl. Phys. **B460** 506 (1996); R. Donagi, B. Ovrut, T. Pantev, and D. Waldram hep-th/0001101 (2000).
15. S. Dimopoulos and F. Wilczek, Report No. NSF-ITP-82-07 (1981), in *The unity of fundamental interactions*, Proc. of the 19th Course of the International School on Subnuclear Physics, Erice, Italy, 1981, Plenum Press, New York (Ed. A. Zichichi); K.S. Babu and S.M. Barr, Phys. Rev. **D48**, 5354 (1993).
16. H. Georgi and C. Jarlskog, Phys. Lett. **B 86**, 297 (1979).
17. S. Barr and C. Albright, hep-ph/0001052 (2000) and these proceedings.
18. S. Barr and S. Raby, Phys. Rev. Lett. **79**, 4748 (1997).